MEASUREMENT UNCERTAINTY

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10/10/2012

Assigning Realistic Tolerances

This paper provides guidelines for assigning realistic tolerances for liquid and gas differential pressure flow measurements. It is based upon the ISO 5167 standard for estimating uncertainties in the measurement of flow rates using differential pressure flow meters. By making some reasonable assumptions for typical configurations it is possible to derive simplified uncertainty estimates for use in, say, data reconciliation.

Measurement Uncertainty

ASSIGNING REALISTIC TOLEARNCES

INTRODUCTION

The output indication of a flow meter, using the differential pressure across the flow measuring device, becomes meaningful only when the accuracy is known. Many sites are introducing data reconciliation and performance reporting software applications which consume large quantities of flow measurement data. The quality of the information produced by these applications is directly related to how well the measurement uncertainties are understood. Presuming that all flow measurement devices have the same measurement uncertainty fails to take into account the differences between gas and liquid measurements, the accuracy of compensation and the certainty of the composition of the stream. Applying the full rigor of ISO 5167 is tedious and requires access to information that is not readily available without considerable detective work.

Therefore, the objective of this paper is to arrive at simplified yet realistic uncertainties by making some generally applicable assumptions with regard to the installed conditions of the flow measurement device for the following typical installations:

- liquid flow with compensation
- liquid flow without compensation
- liquid flow with composition uncertainties
- gases flow with known composition
- gases flow with uncertain composition

SUMMARY OF ISO 5167 FLOW EQUATIONS

In practical installations the mass flow rate, q, through a differential pressure flow meter is taken as being

$$q = K\sqrt{\Delta P}$$

Equation 1

directly proportional to the square root of the differential pressure:

where ΔP is the measured differential pressure

Although K is often taken as a constant, in practice its value changes with stream conditions. The simplest form of correction is compensation for stream conditions:

$$q = \frac{K}{\sqrt{\rho_c}} \sqrt{\rho \Delta P} \text{ -2}$$

Equation 2

where ρ_c is the calibrated density

and ρ is the actual stream density

The 'constant' K can be expressed in terms of the characteristics of the differential measurement device:

Equation 3

where C is the discharge coefficient which varies according to the type of orifice.

and ε is the expansion factor which is a function of β

and d is the throat diameter

and D is the external or bore diameter

and β is equal to d/D.

A flow computer would be able to determine the corresponding value for K, but in practice this is not often used. Therefore Equation 2 is the equation relevant to most plant metering situations.

ESTIMATING FLOW RATE UNCERTAINTIES IN ACCORDANCE WITH ISO 5167

The flow uncertainty can be expressed in either absolute or relative terms:

$$\textit{absolute terms:}\ \ q_a = q+\_\delta q$$

Equation 4a

where δq is the uncertainty expressed in flow units

and q_a is the actual or true mass flow

$$\textit{relative terms:}\ \ q_a = q+\_(\frac{\delta q}{q}).100\%$$

Equation 4b

It should be noted that the uncertainty does not indicate a band within which the actual mass flow lies. Instead it expresses statistically that the actual value will lie within this band with a given certainty, usually 95%.

The uncertainty δq is not known directly, but in terms of the constituent coefficients and values in Equation 2. To calculate δq from this equation we use the following expression:

$$\left(\frac{\delta q}{q}.100\right)^2 = \left(\frac{\delta K}{K}.100\right)^2 + \frac{1}{4}\left(\frac{\delta \Delta P}{\Delta P}.\right)^2 + \frac{1}{4}\left(\frac{\delta p}{p}.100\right)^2$$

Equation 5

where	$\delta K/K \cdot 100$	is the meter coefficient relative uncertainty (%)
and	$\delta \Delta P/\Delta P \cdot 100$	is the differential pressure indication relative uncertainty (%)
and	$\delta \rho/\rho \cdot 100$	is the stream density relative uncertainty (%)

This equation is saying that the relative uncertainty of the flow is a combination of the relative uncertainties of the meter coefficient, the differential pressure and the stream density. Note that it is not a simple summation combination - it is the sum of squares. This arises from the assumption that the uncertainties are statistical quantities (values, drawn from a 'normal' or 'Gaussian' distribution). The 4 factor arises because of the square root term in Equation 3.

Note also that the calibration density does not appear in Equation 5. The argument follows from the assumption that we know with certainty the value used for calibration - even though this value might not correspond to the actual stream density.

Differential Pressure Indication Relative Uncertainty

The uncertainty of the indication of differential pressure, ΔP depends on:

- differential pressure gauge span and range calibration
- transmission errors

- rounding and truncation errors associated with the transmitted numerical value.

To simplify the interpretation of uncertainties the following assumption will be made:

Assumption 1: The differential pressure error is independent of the differential pressure over the range of interest (4:1 turndown).

Since we know that the indicated differential pressure is a function of flow we can rewrite it as follows:

$$\frac{\delta \Delta P}{\Delta P} = \delta \Delta P . \frac{K^2}{q^2} \quad \textit{using Equation 1}$$

$$= \delta \Delta p . \frac{k^2}{q_c^{\,2}} . \frac{q_c^{\,2}}{K^2} . \frac{K^2}{q^2}$$

$$= \frac{\delta \Delta P}{\Delta\, p_c} . (\frac{q_c}{q})^2$$

Equation 6

where q_c is the calibrated or design flow

and ΔP_c is the corresponding design differential pressure (usually 2500 mm H_2O)

Equation 6, says that the differential pressure uncertainty varies inversely with the square of the indicated flow. In other words the effect of a constant differential pressure indication error increases as the flow reduces from the design flow conditions.

Meter Coefficient Relative Uncertainty

The meter coefficient is itself a function of other meter parameters, Equation 3. Therefore to obtain the overall uncertainty requires an understanding of the individual uncertainties associated with each term of Equation 3.

$$(\frac{\delta K}{K}.100)^2 = (\frac{\delta C}{C}.100)^2 + (\frac{\delta \varepsilon}{\varepsilon}.100)^2$$
$$+ 4.[\frac{\beta^4.(1-\beta^4)^{1/2}}{C}].(\frac{\delta D}{D}.100)^2$$
$$+ 4.[1+\frac{\beta^4(1-\beta^4)^{1/2}}{c}].(\frac{\delta d}{d}.100)^2$$

Equation 9

where δC/C·100 is the relative uncertainty of the discharge coefficient

and δε/ε·100 is the relative uncertainty of the expansion coefficient

and δD/D·100 is the relative uncertainty of the bore diameter

and δd/d·100 is the relative uncertainty of the throat diameter

Each of these terms can be assigned 'typical' values assuming installation according to ISO 5167:

Assumption 2:

$$\frac{\delta C}{C}.100 = 0.8\%$$ 9

Assumption 3:

$$\frac{\delta \varepsilon}{\varepsilon}.100 = 4.\frac{\Delta P}{P}\%$$ 10

Assumption 4:

$$\frac{\delta D}{D}.100 = 0.5\% \quad 11$$

Assumption 5:

$$\frac{\delta d}{d}.100 = 0.05\% \quad 12$$

Assumption 6:

$$\beta \leq 0.6 \quad 13$$

Although Assumption 3 suggests a variation of tolerance with the differential-pressure-to-line pressure ratio, this factor is relatively small in comparison with the other terms. Typical values might be ΔP = 100 inches H_2O and P = 5 bar giving a modified Assumption 3:

Assumption 3a:

$$\frac{\delta \varepsilon}{\varepsilon}.100 = 0.2\% \quad 14$$

Stream Density Relative Uncertainty

The treatment of stream density depends on whether it is a liquid or gaseous flow.

Liquid Flows

Stream density uncertainty is highly dependent on the particular service of the instrument. In, say, a refinery the uncertainty of density of 'pure' components is very small (0.03%), but some heavy residual fractions can have significant uncertainty. Rather than assume a constant uncertainty it is recommended that it is assigned according to known stream conditions.

Gaseous Flows

Stream density for a gas is given by the ideal gas law:

$$\delta = \frac{PM}{RT}$$

Equation 10

where P is the stream pressure

and M is the molecular weight

and T is the stream temperature

and R is the universal gas constant at stream conditions and includes compressibility factor).

From this it is possible to derive the uncertainty associated with the gas density:

$$(\frac{\delta p}{p}.100)^2 = (\frac{\delta P}{P}.100)^2 + (\frac{\delta M}{M}.100)^2 + (\frac{\delta R}{R}.100)^2 + (\frac{\delta T}{T}.100)^2$$

Equation 11

where $\delta P/P\cdot 100$ is the uncertainty of the stream pressure

and $\delta M/M\cdot 100$ is the uncertainty of the molecular weight

and $\delta R/R\cdot 100$ is the uncertainty of the compressibility factor associated with the universal gas constant

and $\delta T/T\cdot 100$ is the uncertainty of the absolute temperature of the stream.

In many situations, for example a refinery fuel gas line where the molecular weight of the components varies from hydrogen to ethane or even butane, the uncertainty of the molecular weight dominates.

FLOW MEASUREMENT UNCERTAINTY SUMMARY

Summarizing the terms associated with the flow measurement uncertainty and given the previous assumptions, we achieve the following simplified equation:

$$(\frac{\delta q}{q}.100)^2 = (0.88)^2 + 0.25\frac{q_c^4}{q^4}.(\frac{\delta \Delta P}{\Delta p}.100)^2 + 0.25(\frac{\delta\rho}{\rho}.100)^2$$

Equation 12

where $\delta\rho/\rho\cdot 100$ for liquids is assigned, and for gaseous flow is derived from Equation 11.

With this formula and given some realistic estimates of the differential pressure and stream density uncertainties, a 'reasonable' overall uncertainty can be found at different turndowns of the meter. Even this can be cumbersome - the equation itself is still daunting. What is required is a shortcut method for assigning uncertainties as either percentage relative to the measurement or as a fixed percentage of the instrument range. Equation 12 seems a long way from such a simple interpretation - until it is displayed graphically.

Flow Measurement Uncertainty of Typical Configurations

Each of the accompanying graphs plots the uncertainty, vertical axis, versus flow, horizontal axis. In order to simplify the comparison the graph is plotted for a calibrated flow of 100 units. Therefore the uncertainty is given in either measured units or percentage. A ΔP uncertainty of 2.0% has been used throughout.

Graph 1 is typical of a liquid flow measurement uncertainty when stream condition compensation is used. Superimposed on this graph is a plot of an uncertainty that is assumed to be a fixed percentage of the instrument's span. As can be seen this is a reasonable approximation to that derived in Equation 12. Remember that the uncertainty axes covers a very small range.

Graph 2 is similar to Graph 1 except that no compensation for stream conditions has been assumed. Therefore the density uncertainty term increases to 2%. Even so an uncertainty that is a fixed percentage of the instrument span is a reasonable approximation - as shown by the superimposed plot.

Graph 3 is similar to Graph 2, except that it is assumed that the density varies more. Again a fixed percentage of span is a reasonable approximation to the uncertainty. Note that even if compensation is applied, the stream sample used for the density is likely to be a single, unrepresentative, sample.

Graph 4 is typical of a gas with reasonable certainty of the molecular weight. The uncertainties of the pressure and temperature are assumed to be small, as would be the case if they are under pressure or temperature control. In this case an uncertainty that is a fixed percentage (1.75%) of the instrument span approximates reasonably the uncertainty of Equation 12.

Graph 5 is typical of a gas with substantial composition (molecular weight) uncertainty. In this graph a +/- 10% molecular weight variation has been assumed. In this case the uncertainty throughout the flow range is best represented as 5.5% of the indicated flow, as shown on the superimposed plot.

Graph 6 is similar to Graph 5, but an even greater molecular weight uncertainty (+/- 20%) has been assumed. Again the best uncertainty fit is obtained by assuming a 10.5% of indicated flow.

Figure 1a : Absolute uncertainty of measurement - Liquid flow, Good conditions, With compensation

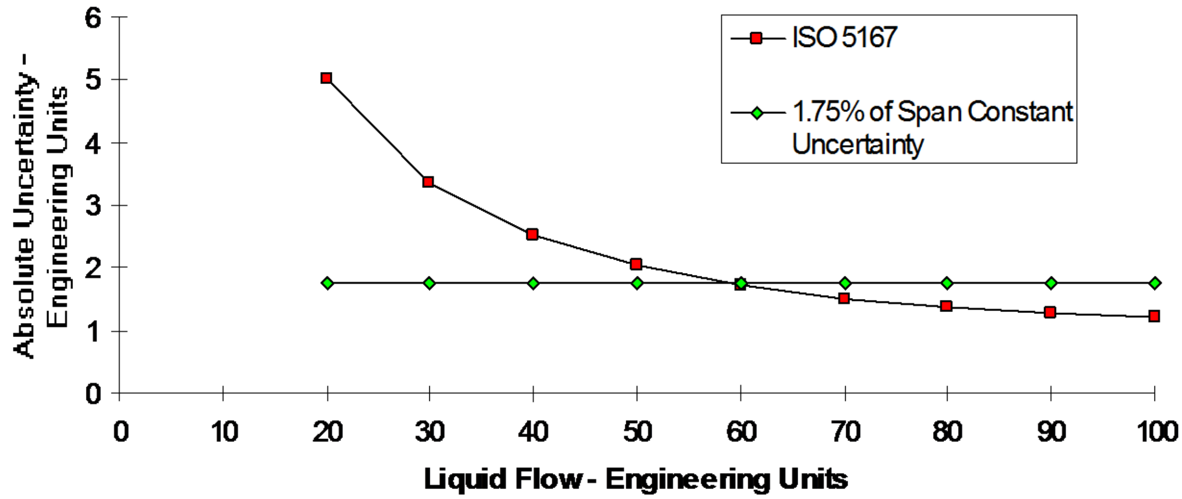
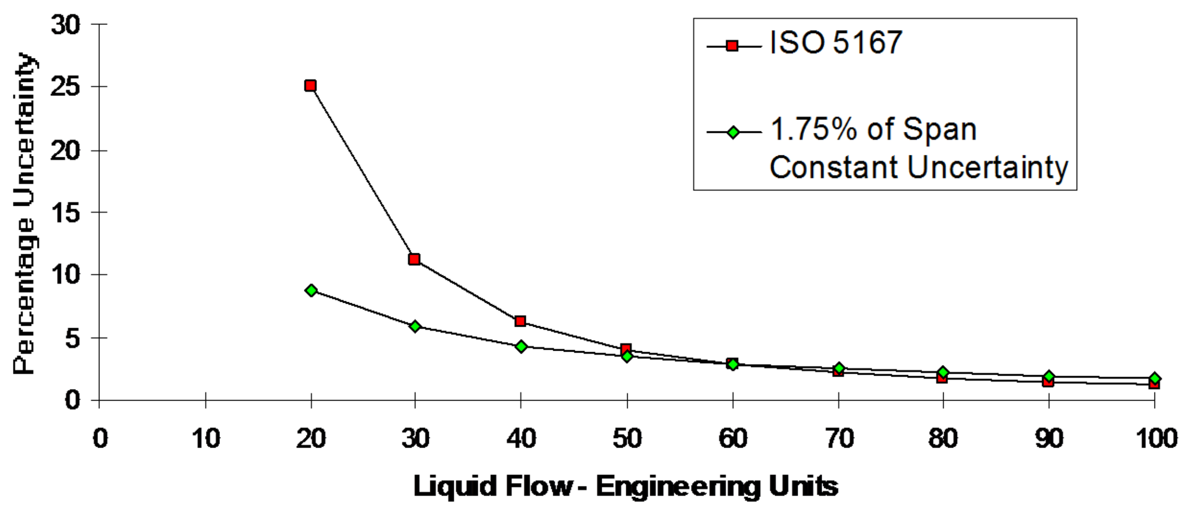


Figure 1b: Percentage uncertainty of measurement - Liquid flow, Good conditions, With compensation



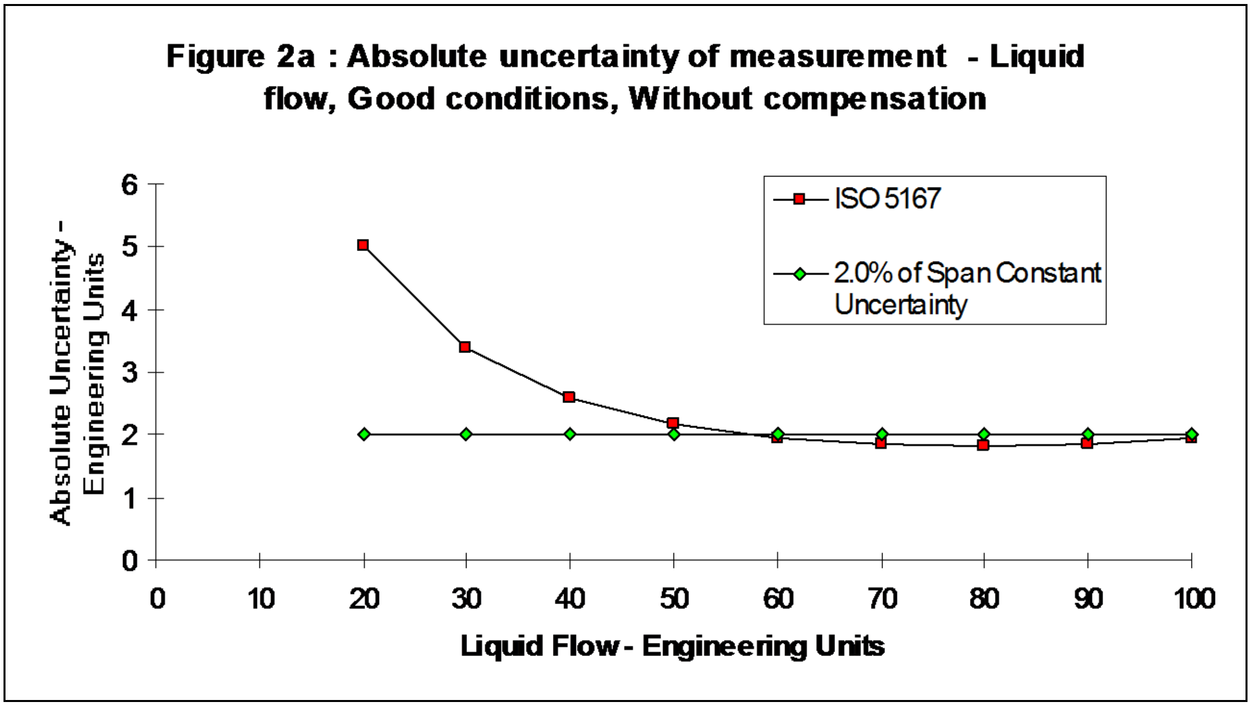


Figure 2a : Absolute uncertainty of measurement - Liquid flow, Good conditions, Without compensation
ISO 5167
2.0% of Span Constant Uncertainty
Absolute Uncertainty - Engineering Units
0
1
2
3
4
5
6
0
10
20
30
40
50
60
70
80
90
100
Liquid Flow - Engineering Units

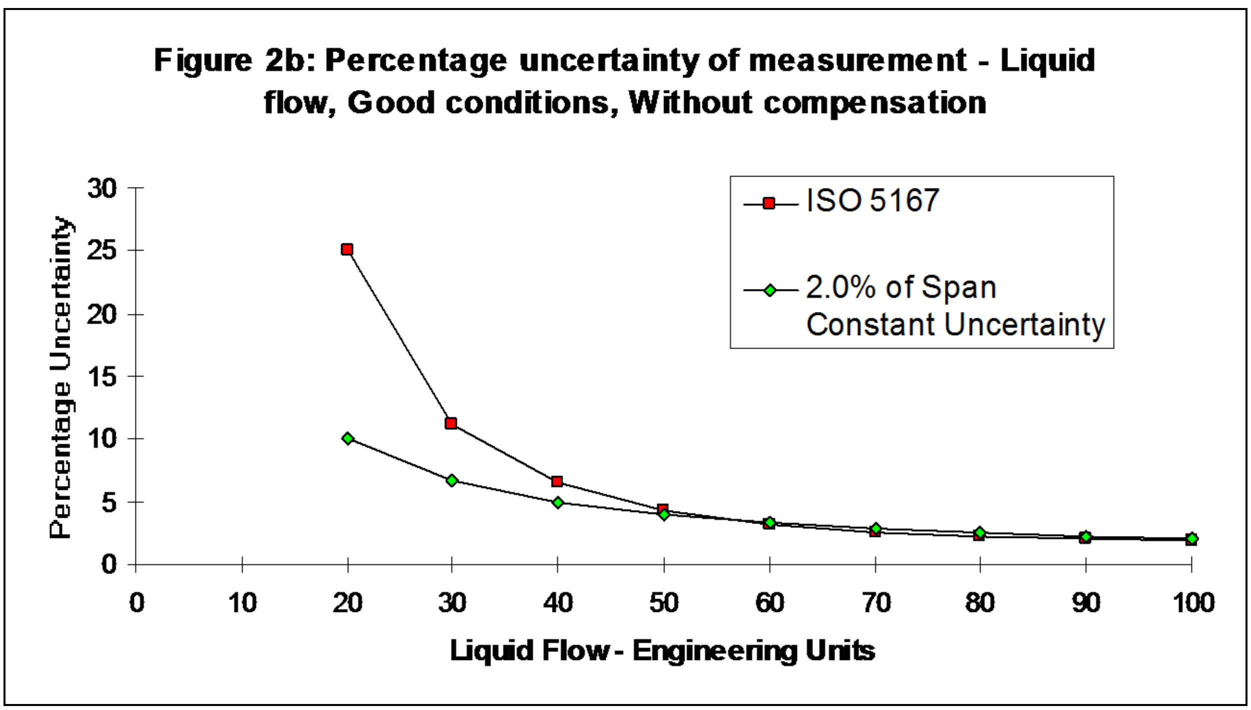


Figure 2b: Percentage uncertainty of measurement - Liquid flow, Good conditions, Without compensation
ISO 5167
2.0% of Span Constant Uncertainty
Percentage Uncertainty
0
5
10
15
20
25
30
0
10
20
30
40
50
60
70
80
90
100
Liquid Flow - Engineering Units

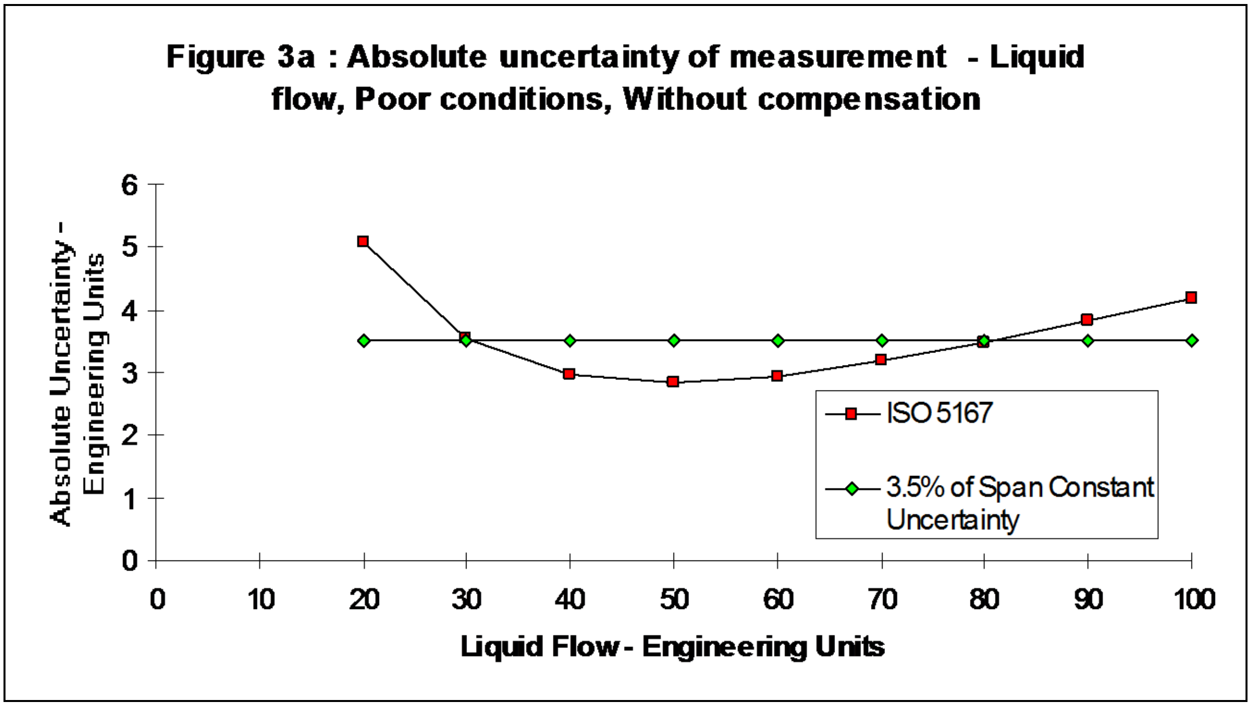


Figure 3a : Absolute uncertainty of measurement - Liquid flow, Poor conditions, Without compensation
Absolute Uncertainty - Engineering Units
Liquid Flow - Engineering Units
ISO 5167
3.5% of Span Constant Uncertainty
0
1
2
3
4
5
6
0
10
20
30
40
50
60
70
80
90
100

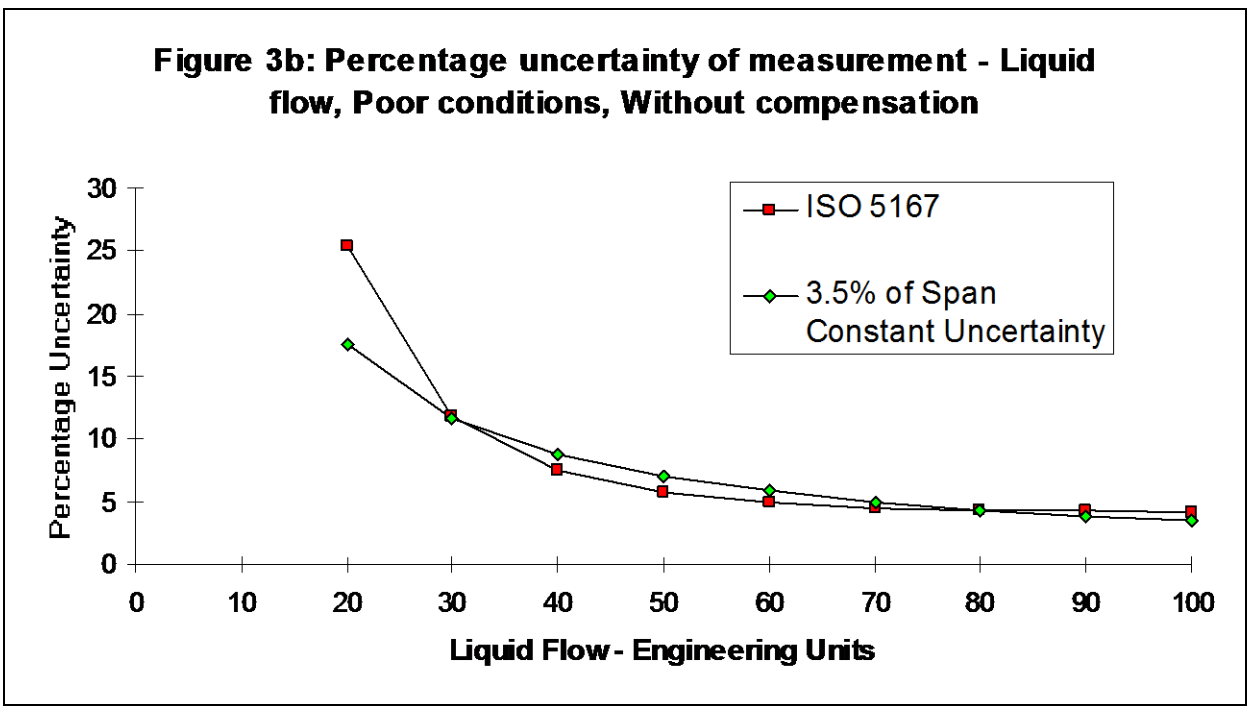


Figure 3b: Percentage uncertainty of measurement - Liquid flow, Poor conditions, Without compensation
Percentage Uncertainty
Liquid Flow - Engineering Units
ISO 5167
3.5% of Span Constant Uncertainty
0
5
10
15
20
25
30
0
10
20
30
40
50
60
70
80
90
100

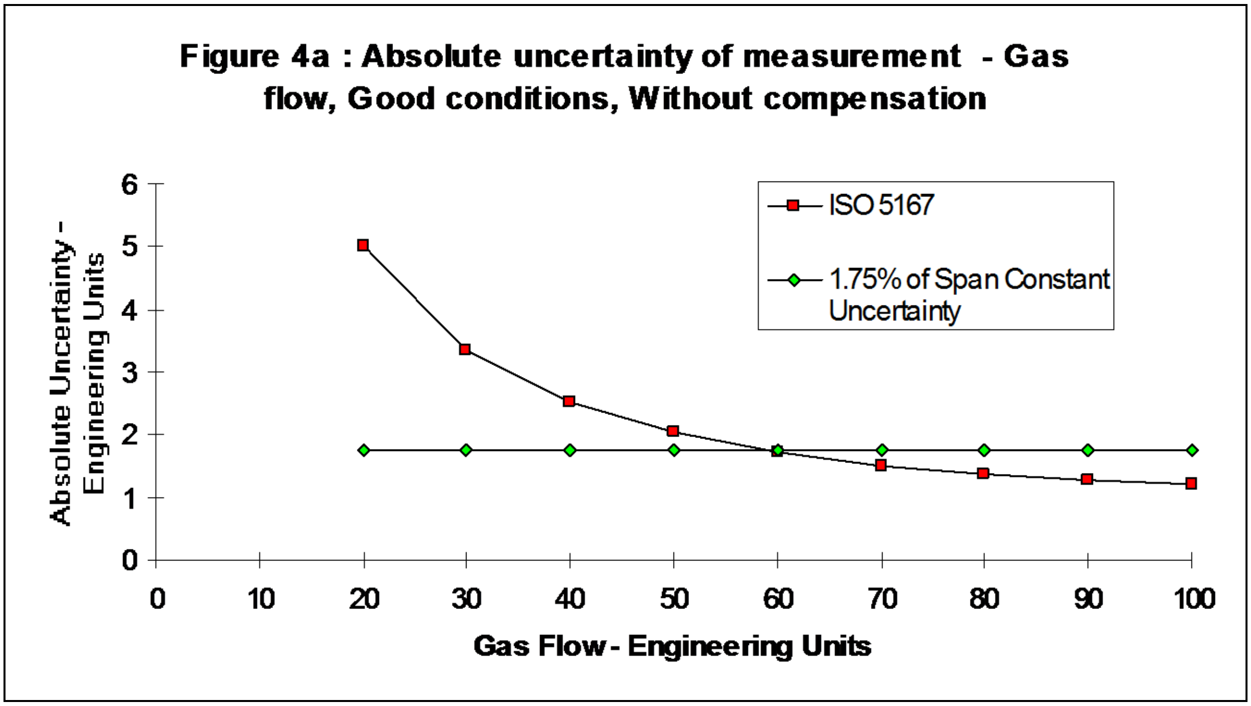


Figure 4a : Absolute uncertainty of measurement - Gas flow, Good conditions, Without compensation
Absolute Uncertainty - Engineering Units
Gas Flow - Engineering Units
ISO 5167
1.75% of Span Constant Uncertainty
0
1
2
3
4
5
6
0
10
20
30
40
50
60
70
80
90
100

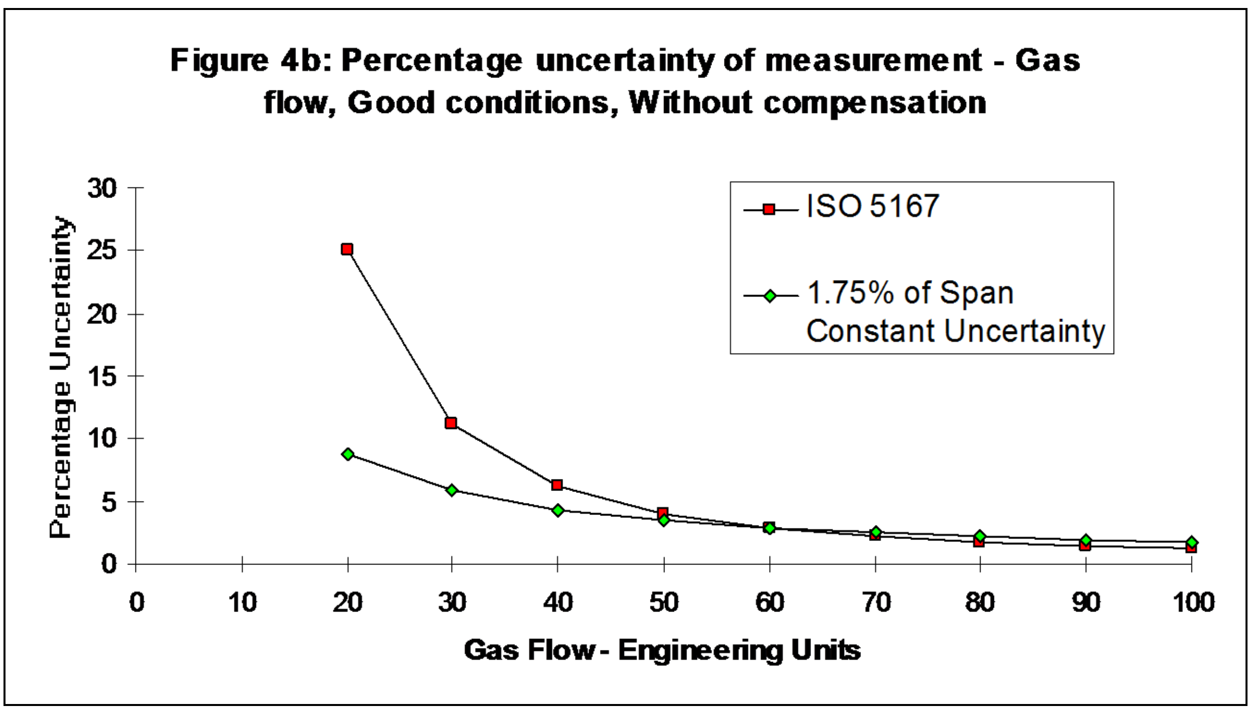


Figure 4b: Percentage uncertainty of measurement - Gas flow, Good conditions, Without compensation
Percentage Uncertainty
Gas Flow - Engineering Units
ISO 5167
1.75% of Span Constant Uncertainty
0
5
10
15
20
25
30
0
10
20
30
40
50
60
70
80
90
100

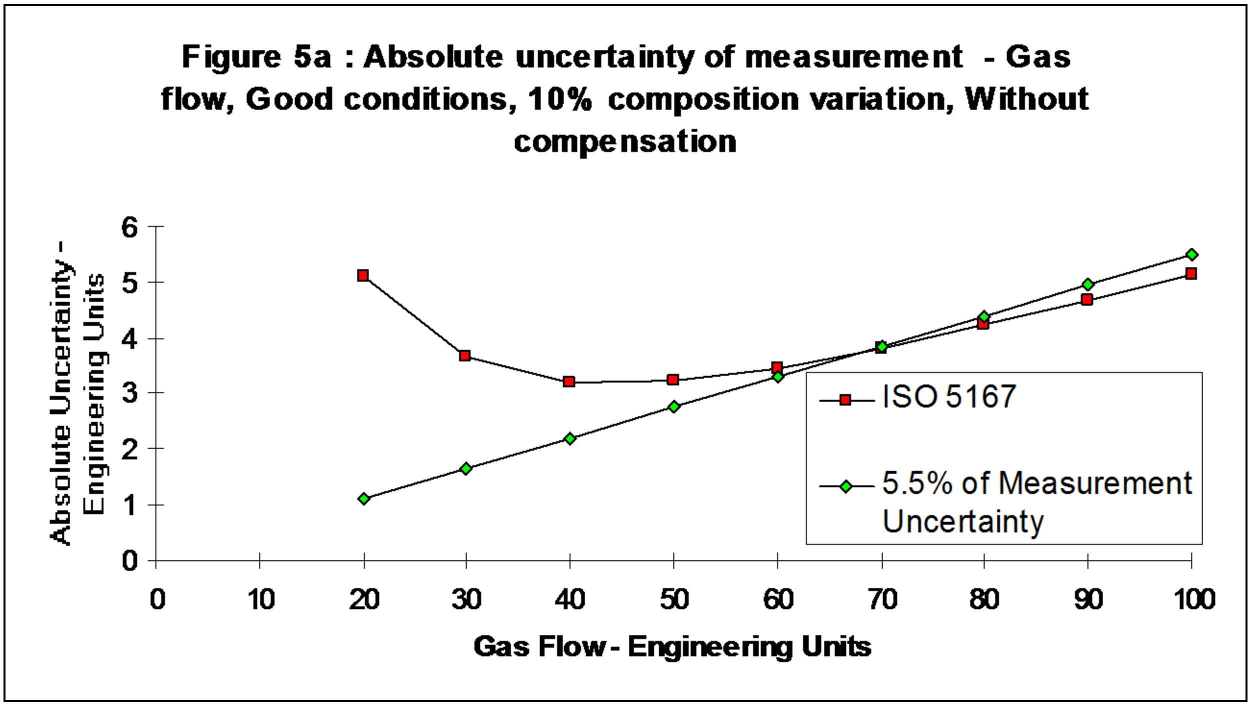


Figure 5a : Absolute uncertainty of measurement - Gas flow, Good conditions, 10% composition variation, Without compensation
Absolute Uncertainty - Engineering Units
Gas Flow - Engineering Units
ISO 5167
5.5% of Measurement Uncertainty
0
1
2
3
4
5
6
0
10
20
30
40
50
60
70
80
90
100

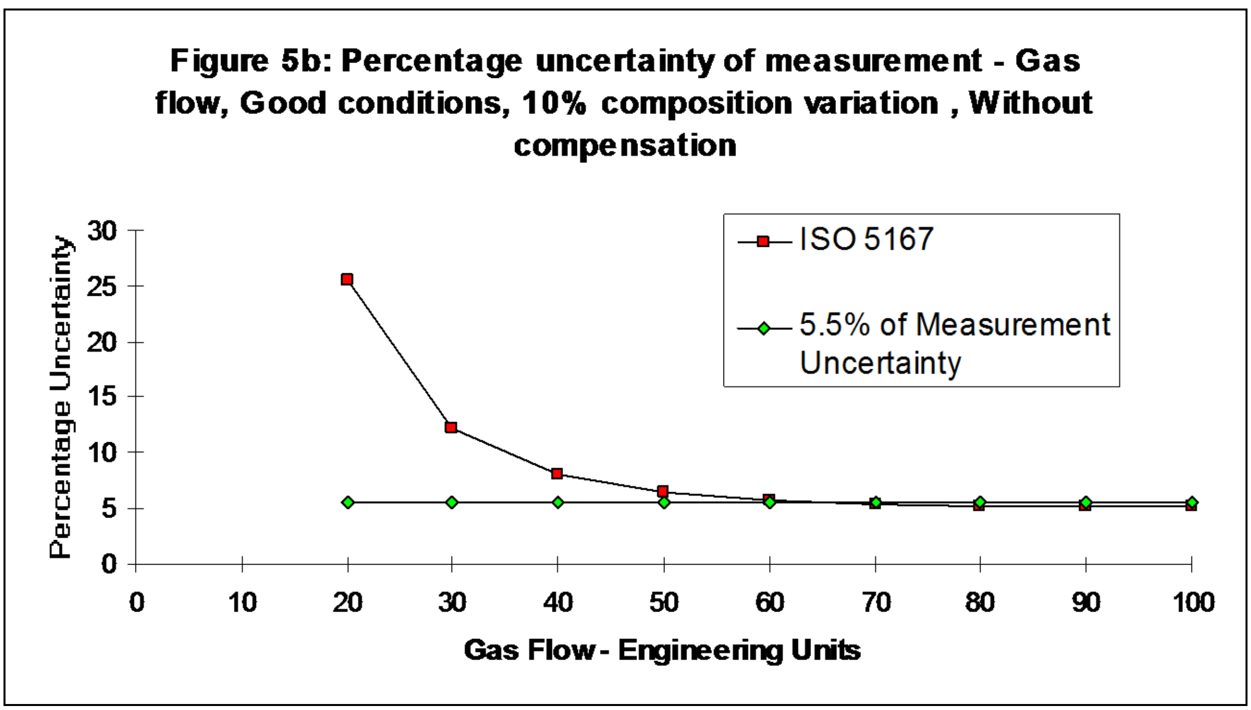


Figure 5b: Percentage uncertainty of measurement - Gas flow, Good conditions, 10% composition variation , Without compensation
Percentage Uncertainty
Gas Flow - Engineering Units
ISO 5167
5.5% of Measurement Uncertainty
0
5
10
15
20
25
30
0
10
20
30
40
50
60
70
80
90
100

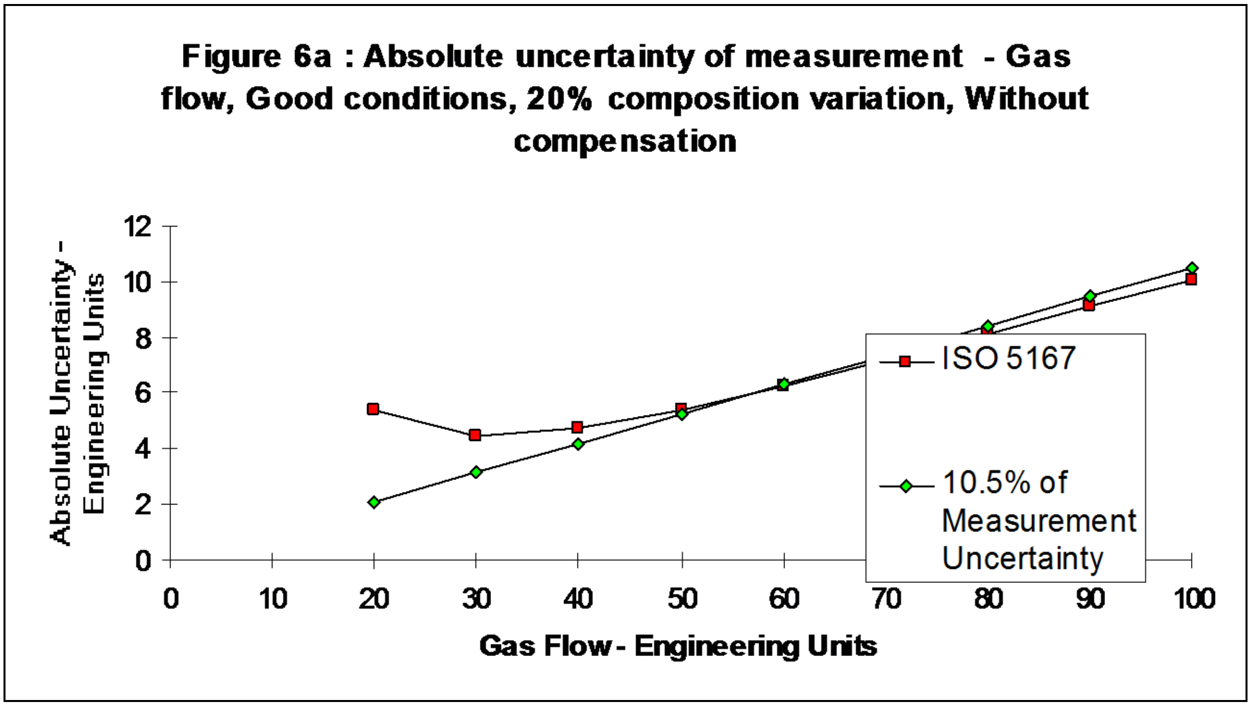


Figure 6a : Absolute uncertainty of measurement - Gas flow, Good conditions, 20% composition variation, Without compensation
Absolute Uncertainty - Engineering Units
12
10
8
6
4
2
0
0
10
20
30
40
50
60
70
80
90
100
Gas Flow - Engineering Units
ISO 5167
10.5% of Measurement Uncertainty

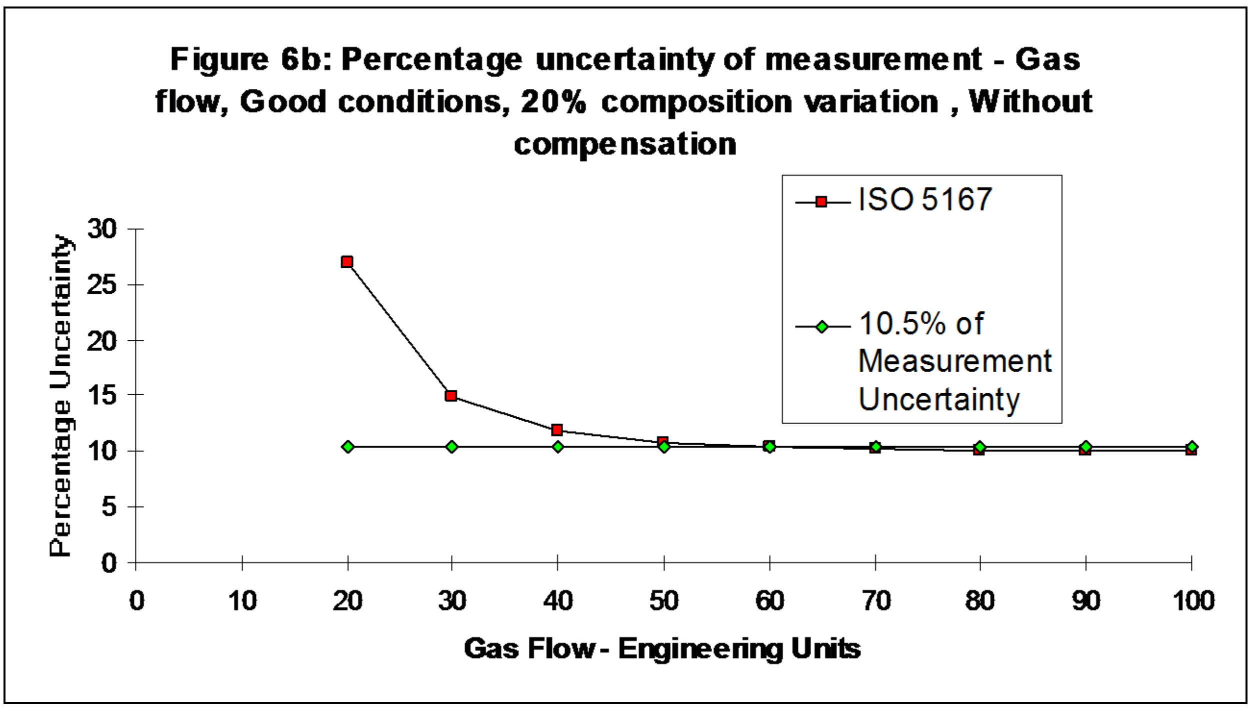


Figure 6b: Percentage uncertainty of measurement - Gas flow, Good conditions, 20% composition variation , Without compensation
Percentage Uncertainty
30
25
20
15
10
5
0
0
10
20
30
40
50
60
70
80
90
100
Gas Flow - Engineering Units
ISO 5167
10.5% of Measurement Uncertainty

CONCLUSIONS

ISO 5167 provides a comprehensive treatment of flow measurements. Unfortunately in the absence of a flow computer, it is difficult to justify the rigor and extent of detective work required to obtain uncertainty estimates. By making some realistic assumptions for typical installations it is possible to reduce the uncertainty to be either a fixed percentage of span or fixed percentage of measurement. The latter is used for gas flow measurement with variable composition. The former should be used in most other circumstances.

Table 1: Flow Measurement Uncertainties

	Characteristics	**Compensation**	**Density or Composition Uncertainty**	**Uncertainty**	
				% Span	**% Measurement**
Liquid	• Good Conditions • Compensation • Stable Composition	Yes	+/- 0%	+/- 1.75	
	• Good Conditions • No Compensation	No	+/- 3%	+/- 2.0	
	• Poor Conditions • Variable Composition	Yes	+/- 8%	+/- 3.5	
Gas	• Good Conditions • Known Composition	No	+/- 0%	+/- 1.75	
	• Good Conditions • Variable Composition	No	+/- 10%		+/- 5.5
	• Reasonable	No	+/- 20%		+/- 10.5

	Conditions • Highly Variable Composition				